

How a group should be divided into subgroups for efficient division of labor

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1 Aim

Condorcet's jury theorem states that the group decision accuracy of a group consisting of N members is an increasing function of N when each member makes a decision correctly with probability exceeding one-half. This theorem deals with the entire group's decision making on a single issue. In the real world, however, a group facing multiple tasks often assigns the tasks to subgroups to obtain collective benefits efficiently. The aims of this study are to show that Condorcet's jury theorem can be applied to an analysis of efficiency of the division of labor and to find a group composition maximizing collective benefit.

2 Model

For those purposes, I constructed a mathematical model. Each member makes a correct decision with probability p for any tasks. The entire group has to treat m tasks. The entire group is divided into m subgroups to deal with those tasks in parallel. Each subgroup makes a group decision using simple majority rule. If a subgroup reaches a correct decision, the subgroup obtains the benefit, which is set to 1. The collective benefit is expressed as the sum of subgroup's benefits.

3 Results

Using Lagrange multipliers method, I found that when members' competence p is greater (smaller) than one-half, dividing the entire group into equal-sized subgroups maximizes (minimizes) the collective benefit. I also considered a model in which each decision-making incurs cost to examine whether saving time with the division of labor more than compensate for the deterioration of subgroups' decision accuracy. As a result, if $p < 0.5$, division of labor always outperforms the plenary voting. When $p > 0.5$, if decision-making cost is greater than around $1 - p$, division of labor is more beneficial than the plenary voting.

4 Conclusion

I showed the conditions for efficient division of labor in the framework of Condorcet's jury theorem. My model assumes that all members have the same competence. The present study should provide a foundation for examining more complicated situations.

References

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